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# APPLICATION
# FOR
# UNITED STATES
# LETTERS PATENT

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# SYSTEM AND METHOD FOR BANDWIDTH MANAGEMENT, PRICING, AND CAPACITY PLANNING

## DESCRIPTION

## BACKGROUND OF THE INVENTION

### *Field of the Invention*

The present invention generally relates to buying and selling bandwidth and, more particularly, to a system which combines chance constrained programming with variable pricing as a tool for bandwidth management.

### *Description of the Related Art*

The bandwidth of a transmitted communications signal is a measure of the range of frequencies the signal occupies. All transmitted signals, whether analog or digital, have a certain bandwidth. As large communications companies expand the capabilities of their current systems with vast new high-speed networks to meet projected future demands, they inevitably create surplus bandwidth in the present.

Bandwidth is traded like a commodity or security. The current growth of the bandwidth market, driven by increasing Internet use and electronic commerce, is running somewhere between 25% and 40% a year. By 2005, it has been predicted by some that the bandwidth trading market in the U.S. may be

more than $400 billion.

Consider a re-seller who buys surplus bandwidth in bulk from a large communication company and resells it in smaller bundles to customers. These bundles correspond to "contracts" made with customers to supply bandwidth in standard quantities for a specific time span. The type of contract bought defines a "customer class" or "customer type".

A customer class $i$ is *defined* by the parameters $\mu_i$ and $\sigma^2_i$ which are the mean and variance of the (normal) distribution describing this class of user's consumption of bandwidth.

## SUMMARY OF THE INVENTION

A question arises as to how the reseller should price these contracts. The reseller must not only choose prices which will attract customers, but also make sure that these customers do not collectively exceed the bandwidth available (i.e. sold). Since the behavior of the end users is neither deterministic nor under the re-seller's control, we shall take this to mean that given the distribution of individual customer bandwidth consumption, the total available shall not be exceeded with some (high) probability at any time $t$ within the planning horizon. This is accomplished by means of "chance-constraining" total bandwidth consumption.

The present invention is directed to a system which

combines a chance constrained optimization model with variable pricing as a tool for bandwidth management. Performance analysis and capacity planning are integrated with the pricing scheme. This is a discretized multi-time-period model, where the time $t$ is specified in terms of multiples $\tau$ of a fixed period length $\Delta$.

## BRIEF DESCRIPTION OF THE DRAWINGS

The foregoing and other objects, aspects and advantages will be better understood from the following detailed description of a preferred embodiment of the invention with reference to the drawings, in which:

Figure 1 is a flow diagram showing the data acquisition and input steps according to the present invention; and

Figure 2 is a flow diagram showing the optimization and output steps for determining a price structure for contracts offered to clients.

## DETAILED DESCRIPTION OF A PREFERRED EMBODIMENT OF THE INVENTION

A "chance constraint" is an inequality on the variables in the model that must be satisfied with some probability less than 1, in contrast to ordinary constraints, which must be completely satisfied (i.e. with probability 1).

We illustrate this approach considering only one class of customer. For any fixed time $t$, let



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$$Y_t = X_1 + X_2 + \cdots + X_{N(t)}$$

where (we assume) the $X_i$'s are identical independent normal distributed random variables with mean $\mu$ and variance $\sigma^2$, which represent the real usage of $N(t)$ customers. $Y(t)$ is then the random variable representing the total bandwidth consumption of these $N(t)$ customers at time $t$. Further assuming that customers arrival is described by a Poisson distribution with $\lambda = \lambda(t)$, independent of $X_i$, then:

$$P(N(t)=k) = [\frac{(\lambda(t))^k e^{-\lambda(t)}}{k!}].$$

The parameter $\lambda(t)$ will be related to the price set, as we shall discuss below.

To specify that the customers' collective bandwidth consumption $Y(t)$ does not exceed the available bandwidth $b_t$, with some (high) probability $\delta_t$, we impose the chance constraint:

$$P(Y_t > b_t) \leq \delta_t,$$

The invention requires that this chance constraint be expressed in a computationally tractable way. This is carried out using standard techniques from probability theory (see e.g. W. Feller, *An Introduction to Probability*



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*Theory and its Applications*, Vol 1 (3rd edition), (1968) and Vol 2, Wiley, NY (1971)) as follows:

Define the moment generating function for $Y_t$:

$$\psi_r(Y_t)=E[e^{rYt}],$$

and note that $E[Y_t^2]$ can be derived from this moment generating function via the relation:

$$E[Y_t^2]=\frac{\partial^2\psi_r(Y_t)}{\partial r^2}\Big|r=0.$$

which on differentiating, yields:

$$E[Y_t^2]=\lambda(t)\sigma^2+\lambda(t)\mu^2+(\lambda(t))^2\mu^2.$$

Applying the Chebyshev bound we also derive:

$$P(Y_t>b_t)\le\frac{E[Y_t^2]}{b_t^2}$$

hence, using our expression for $E[Y_t^2]$, we see that the chance constraint is satisfied if:

$$\lambda(t)\sigma^2+\lambda(t)\mu^2+(\lambda(t))^2\mu^2 \le \delta_t b_t^2.$$

This derivation may be generalized to multiple customer classes and multiple discrete time periods, and is applied in the most general form of the invention.

Referring now to the drawings, and more particularly to Figure 1 there is shown a flow diagram showing the data acquisition and input steps according to the present invention for optimizing bandwidth management with multiple types of contracts.

Below are listed the notations, assumptions, and data for implementing the present invention.

*Indices*

$i = 1, \ldots, I$: customer class;

$\tau = 1, \ldots, T$: time periods, each of length $\Delta$.

*Assumptions*

For any fixed time $t$, real usages of signed-on customers for class $i$ are identical independent normal distribution with mean $\mu_i(t)$ and variance $\sigma_i^2(t)$;

Number of customers of class $i$ is Poisson with parameter $\lambda_i$, itself a function of price (see below).

*Data*

$\delta_\tau$: tolerance on capacity violation in period $\tau$;

$C_\tau$: cost per unit of buying new capacity in period $\tau$;

$d_\tau$: duration of contract (number of time periods) for customer class $i$;

$D_i$: actual duration of contract ($d_i\Delta$) for customer class $i$;

$n_{i\tau}$: number of existing contracts of type $i$ still active at start of period $\tau$;

$L_{i\tau}$: lower bound on contract price;

$U_{i\tau}$: upper bound on contract price.

*Variables*

$b_\tau$: the bandwidth available in period $\tau$ (non-negative);

$a_\tau$: bandwidth purchased by re-seller in period $\tau$ (non-negative);

$q_{i\tau}$: price to new (or renewing) customers for a new standard length contract of type $i$ in period $\tau$.

*User Supplied Functions*

$\lambda_i(q_{i\tau})$: the expected number of new customers of type $i$ arriving in any period if the price for a contract is set at $q_{i\tau}$.

These are standard price-demand curves reflecting the elasticity of demand.

*Constraints*

In addition to the constraint on the availability of the bandwidth at each time $\tau$ and on the price range, it is required that the total available shall not be exceeded with some (high) probability at any time $t$ within the planning horizon:

$$\boldsymbol{b}_\tau = \boldsymbol{b}_{\tau-1} + \boldsymbol{a}_\tau \qquad (\tau = 1,...,T) \qquad (1)$$

$$L_{i\tau} \le q_{i\tau} \le U_{i\tau} \qquad (i = 1,..., I;\ \tau = 1,..., T) \qquad (2)$$

$$\sum_{i|\tau<d_i} [\lambda_{i\tau}\Delta\mu_i^2 + (n_{i\tau} + \lambda_{i\tau}\Delta)^2 \sigma_i^2 + (n_{i\tau} + \lambda_{i\tau}\Delta)^2\mu_i^2]$$

$$+ \sum_{i|\tau\geq d_i} [\lambda_i D_i(\mu_i^2 + \sigma_i^2)] + (\lambda_i D_i \mu_i)^2] - \delta_\tau b_\tau^2 \leq 0 \quad \forall\tau \qquad (3)$$

The constraint (3) is a deterministic expression of the requirement that:

$Pr\{$(Bandwidth consumed by customers at time $t$) $> b_t) \leq \delta_t$

This transformation is carried out by a generalization of the process as set forth starting in the first paragraphs of the detailed description section for the single customer class, single time period case. (Explicit details of this generalization are given in a technical report by the inventors of the present invention - IBM Research Report RJ 10196 (95070) November 2, 2000), herein incorporated by reference.

*Objective*

The objective of the present invention is to maximize the total revenue minus the purchase cost

$$\text{Maximize} \sum_{i,\tau} q_{i\tau}\lambda_i(q_{i\tau}) - \sum_{\tau} C_\tau a_\tau \qquad (4)$$

Referring now to Figure 1 there is shown a flow diagram showing the data acquisition and input steps according to the present invention for optimizing bandwidth management with multiple types of contracts.

In box 10 the mean and variance of the real usage of each customer class is obtained. In box 12, the price-demand curve data which determines the arrival rate for each customer class is obtained. In box 14, the data on the number of existing customers in each class is obtained. Finally, in box 16, the the bandwidth wholesale cost to the reseller and other items specified in the "Data" section above is obtained.

Figure 2 is a flow diagram showing the optimization steps according to the present invention. In box 20, a computer model is generated which embodies the objective (4) and the constraints (1, 2, 3). Thereafter, in box 22, nonlinear programming software is used to solve the optimization problem. For example, the MINOS nonlinear optimizer available from Stanford Business Software, Inc. is an example of a suitable software package for this model.

Finally, in box 24, based on the non-linear programming solution, design a price structure for the contracts offered to customers. The prices are obtained explicitly from the

values of the $q_{it}$ variables in the optimal solution, giving prices to be charged by customer class and time period.

**Example:**

As an example, let us take a simple case with a single time period, single customer class, and fixed contract duration, starting at time 0, the available capacity is 10 units. We need to decide how much bandwidth to purchase at time 0 to satisfy demand during the time period [0,1]. Using arbitrary units, assume the average usage $\mu$ of this customer class is 2 units, with variance $\sigma^2 = 1$. Also, we assume that it costs 1K dollars to buy each unit of bandwidth. We also specify that at time 0, the initial available bandwidth $b_0$ is 10 units, and that the tolerance level is $\delta = 0.99$. We now need to choose an optimal purchase plan and pricing scheme to maximize our profit.

We recall that price and demand are assumed dependent on each other, and start with the simplest case in which we are to choose between two options:

(A): if we charge each new customer 2.5K dollars, the expected customer number $\lambda$ will be 20;

(B): if we charge each new customer 2K dollars, the expected number $\lambda$ will be 30.

Now, which price scheme should be adopted? A or B? In consequence, how much bandwidth $a$ should we purchase to meet

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the 0.99 tolerance criterion?

Formulating the simplified optimization problem and substituting the given numerical values for the case A in optimization for constraints (1), (2), and (3):

Maximize

$$20 \times 2.5 - a$$

subject to

$$20 \times 4 + 20 \times 1 + 20 \times 20 \times 4 - 0.99(10 + a)^2 \leq 0$$

and obtain the solution: $a = 32$, $b_1 = b_0 + a = 42$, and the profit is 18K dollars.

Solving the optimization problem for case B:
Maximize

$$30 \times 2 - a$$

subject to

$$30 \times 4 + 30 \times 1 + 30 \times 30 \times 4 - 0.99(10 + a)^2 \leq 0$$

we obtain the solution: $a = 52$, $b_1 = b_0 + a = 62$, and the profit is 8K dollars.
Clearly, plan A provides more profit.

In general, demand is continuously sensitive to the price charged, not just for two possibilities A and B as above, and demand and price are believed to be reversely

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correlated. Taking the above example, and if we assume that price and demand are linearly dependent, then we have

$$\lambda_1 = \lambda_1(q_{11}) = 70 - 20q_{11}$$

or equivalently:

$$q_{11}(\lambda_1) = \frac{70-\lambda}{20}$$

where $\lambda_1$ is the number of customers and the $q_{11}$ is the price charged to each customer (A and B are 2 special cases of this).

Now, we need to choose the best $q_{11}$ and $a_1$ to maximize our profit. The constraints, are

$$b1 = 10 + a1 \qquad (1)$$

$$0 \leq q11 \qquad (2)$$

$$5\lambda1 + 4\lambda12 - 0.99b12 \leq 0 \qquad (3)$$

and the objective is

$$\max_{\lambda\leq 0, a\geq 0} q_{11}(70-20q_{11}) - a_1 \qquad (4)$$

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This can be solved explicitly (in this case using nonlinear optimizer software, such as MINOS) to obtain the optimal solution:

$a = 21.15$, $\lambda = 14.88$ and the profit is 19.87K dollars.

According to the present invention capacity planning and pricing policy are directly related to and reflective of the demand of the market. Under the proposed pricing policy, it is in the best interest of the customers to choose the level of the bandwidth service that best reflects their real demand; hence, the capacity planning is more efficient. Further, the pricing scheme provides more flexible choice of the bandwidth service level, allowing certain range of variance. Since the pricing policy is directly related to the real performance of the bandwidth service level, this model provides better control over possible bursts and helps to improve bandwidth management for both the company and the customer. The present invention is preferably implemented in software and of course may comprise computer instructions on a computer readable medium such as a disk, tape, chip or the like.

While the invention has been described in terms of a single preferred embodiment, those skilled in the art will recognize that the invention can be practiced with modification within the spirit and scope of the appended claims.



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